A Non-Technical Introduction to the Evaluation of Informative Hypotheses using Bayesian Model Selection

Rens van de Schoot, Joris Mulder,

Herbert Hoijtink

Department of Methodology and Statistics

Utrecht University, The Netherlands

Marcel A. G. Van Aken, Judit S. Dubas, Bram Orobio de Castro

Department of Developmental Psychology

Utrecht University, The Netherlands

Wim Meeus

Research Centre Adolescent Development

Utrecht University, The Netherlands

Jan-Willem Romeijn

Department of Philosophy

Groningen University, The Netherlands

Abstract

In practice, it is often the case that a researcher has expectations about their research questions that can be formulated in terms of informative hypotheses. For example, the mean of group 1 is larger than the mean of group 2 and group 3, but smaller than group 4. Bayesian model selection (BMS) can be used to evaluate such informative hypotheses using Bayes factors as selection criteria. By now, a wide variety of models specified with (in)equality constraints can be analyzed using BMS. Although BMS has been described in previous articles, these papers are rather technical and published solely in statistical journals. The main objective of this article is to provide an easy to read introduction to BMS. Moreover, we provide a two-step procedure how to interpret the results of BMS. This is illustrated using an example from psychology.

Keywords: Bayesian model selection, inequality constraints, informative hypothesis, Bayes factors.

A Non-Technical Introduction to the Evaluation of Informative Hypotheses with Bayesian Model Selection

Null hypothesis testing has been the dominant research tool in the social and behavioural sciences over the latter half of the past century. A valuable alternative for testing the null hypothesis is evaluating *informative* hypotheses using Bayesian Model Selection (BMS) (Hoijtink, Klugkist, & Boelen, 2008). We call a hypothesis *informative* if it contains information about the relationship between model parameters. More specifically, model parameters such as mean scores or regression coefficients can be constrained to being greater or smaller than either a fixed value or other statistical parameters. An example of an informative hypothesis is that the mean of a variable of group 1, is larger than the mean of group 2, which in turn is smaller than group 3. Evaluation of informative hypotheses using BMS is emerging in the psychological literature (Boelen & Hoijtink, 2008; Laudy, et al., 2005; Meeus, Van de Schoot, Keijsers, Schwartz, & Branje, 2009; Van de Schoot, Hoijtink, & Doosje, 2009; Van de Schoot, & Wong, 2008; Van Well, Kolk, & Klugkist, 2008).

Previous articles about the methodology mainly deal with the technical aspects (for an overview see: Hoijtink et al., 2008). The purpose of this article is therefore to (i) present an easy to read introduction to BMS for applied researchers, and (ii) to provide guidelines how to interpret the results of BMS. This is necessary because, unlike classical hypothesis testing, BMS does not use *p*-values, but Bayes factors. These are calculated for each informative under evaluation and provide the amount of support from the data for each hypothesis. The methodology is illustrated using a study from developmental psychology.

Example

Van Aken and Dubas (2004) investigated differences between three personality types in adolescence: resilient (R), over-controlled (O), and under-controlled adolescents (U). The main question was whether psychosocial functioning (externalizing (E), internalizing (I) and social problem (S) behavior) is the result of the interplay between personality and support from family.

Just like in the original article, personality type was assessed by big-five personality markers (Gerris, Houtmans et al., 1998). Furthermore, the problem behaviour list (PBL; De Bruyn, Vermulst, & Scholte, 2003) was used to obtain parent reports on children's behavioural problems, measured on a 5-point scale. Finally, the relational support inventory (RSI; Scholte, Van Lieshout, & Van Aken, 2001) was used to measure the support that children receive from their parents to obtain high versus low family support.

Based on personality type (R, O, U), high or low family support (H, L), 3 x 2 = 6 groups of adolescents were constructed assessed on three dependent variables (E, I, S), see Table 1. Let μ denote the mean score on the dependent variable, for example μ_{RHE} is the mean score for Resilient adolescents with High family support on the dependent variable Externalizing behavior. Below, three expectations are presented concerning the ordering of these 6 groups. Such expectations are what we call informative hypotheses.

The first two expectations (H_A and H_B) are based on several studies showing that the three personality types have a distinct pattern of psychosocial and relational functioning (see, for example, Van Aken, Van Lieshout, Scholte, & Haselager, 2002). The first expectation (H_A) states that under-controllers are expected to have the most externalizing problems and over-controllers are expected to have the most internalizing problems. Over-controllers (O) and under-controllers (U) are believed to score higher on social problems compared to resilient adolescents (R). Moreover, no constraints are specified with respect to high/low family support. The informative hypothesis H_A can be formulated as

$$H_A: \begin{cases} (\mu_{RHE}, \mu_{RLE}, \mu_{OHE}, \mu_{OLE}) < (\mu_{UHE}, \mu_{ULE}) \\ (\mu_{RHI}, \mu_{RLI}, \mu_{UHI}, \mu_{ULI}) < (\mu_{OHI}, \mu_{OLI}) \\ (\mu_{RHS}, \mu_{RLS}) < (\mu_{OHS}, \mu_{OLS}, \mu_{UHS}, \mu_{ULS}). \end{cases} \tag{1}$$

The second expectation (H_B) states, additionally to H_A, that resilient adolescents best function in all psychosocial domains in comparison to the other two types of adolescents. Hence, the informative hypothesis H_B contains two additional constraints in comparison to H_A,

$$H_B: \quad \begin{array}{l} (\mu_{RHE}, \mu_{RLE}) < (\mu_{OHE}, \mu_{OLE}) < (\mu_{UHE}, \mu_{ULE}) \\ (\mu_{RHI}, \mu_{RLI}) < (\mu_{UHI}, \mu_{ULI}) < (\mu_{OHI}, \mu_{OLI}) \\ (\mu_{RHS}, \mu_{RLS}) < (\mu_{OHS}, \mu_{OLS}, \mu_{UHS}, \mu_{ULS}). \end{array} \tag{2}$$

Previous research also indicates that it is the combination of personality type and the quality of social relationships that determines the risk level for experiencing more problem behavior (Van Aken et al., 2002). Therefore, additional constraints are constructed for the third expectation (H_C). Over- and under-controllers with high perceived support from parents are expected to function better in psychosocial domains than those with low perceived support. For the resilient group, the level of support from parents is not related to problem behavior. The additional constraints for informative hypothesis H_C are

$$H_C: \begin{cases} (\mu_{RHE} = \mu_{RLE}) \, , \, (\mu_{OHE} < \mu_{OLE}) \, , \, (\mu_{UHE} < \mu_{ULE}) \\ (\mu_{RHI} = \mu_{RLI}) \, , \, (\mu_{UHI} < \mu_{ULI}) \, , \, (\mu_{OHI} < \mu_{OLI}) \\ (\mu_{RHS} = \mu_{RLS}) \, , \, (\mu_{OHS} < \mu_{OLS}) \, , \, (\mu_{UHS} < \mu_{ULS}). \end{cases} \tag{3}$$

Now that the research expectations are carefully formulated, the goal is to determine which of hypothesis H_A or H_B receives most support from the data. The best of these two hypotheses is then combined with the constraints of H_C to investigate whether these additional constraints are supported by the data.

The Method of Bayesian Model Selection

The method of BMS provides an answer to the research question which informative hypothesis receives most support from the data. This is described step by step in this section.

Simple Example

To explain BMS, consider the following simple example. Suppose the research question is whether over- (O) and under-controlled (U) adolescents differ on externalizing behavioral problems. Furthermore, suppose the first hypothesis (H_A) postulates that there is no restriction between the means on externalizing behaviour, that is, any combination of means is admissible. This model is also known as the unconstrained hypothesis. The second hypothesis (H_B) postulates that the externalizing behavioral problems of both groups are equal. This model is also known as an equality constrained hypothesis. The third hypothesis (H_C) postulates that over-controlled adolescents score lower on externalizing problem behavior than under-controlled adolescents. This model is also known as an inequality constrained hypothesis. Formally, the three hypotheses in this simple example are

$$\begin{aligned} &H_A: \mu_O\ ,\ \mu_U; \\ &H_B: \mu_O = \mu_U; \\ &H_C: \mu_O < \mu_U. \end{aligned} \tag{7}$$

To evaluate the set of hypotheses in (7) with BMS, three components are needed that will be explained successively: (1) admissible parameter space, which represents the expectations of the researcher; (2) the likelihood of data, which represents the information in the data set with respect to μ_O and μ_U ; and (3) the marginal likelihood, which represents the support from the data for each hypothesis, combining model fit and model size. This latter component can be converted in Bayes a factor, which is the model selection criterion used in our methodology and combines the above components.

Admissible Parameter Space

The first component is the admissible parameter space, which is based on the (in)equality constraints in the informative hypothesis. Let the squares in Figure 1 represent the complete parameter space. For H_A, every combination of μ_O and μ_U is permitted, and therefore, the admissible parameter space of H_A is equal to the total parameter space (Figure 1). For H_B, μ_O and μ_U must be equal, which implies that only that part of the parameter space is admissible in which μ_O is equal to μ_U. This is represented by the diagonal in Figure 1. For H_C, only combinations of μ_O and μ_U are permitted in which μ_O is smaller than μ_U, which results in the lower triangle in Figure 1. Thus, the admissible parameter space is the total of all possible combinations of the two means for μ_O and μ_U that satisfy the restrictions of each of the hypotheses (H_A, H_B, H_C) before observing the data. In sum, with respect to admissible parameter space, the hypotheses can be ordered from a small parameter space to a large parameter space: H_B, H_C, H_A.

The actual specification of prior distributions in this situation is far from easy and is not considered the topic of this paper. The interested reader is referred to Mulder, Hoijtink, and Klugkist (2009) for a detailed description of a default specification of the prior used in the software.

Likelihood of the Data

The second component is the likelihood of the data, which is the representation of the information about the means in the data set. In Figure 2 the likelihood function is plotted as a function of μ_O and μ_U. The higher this surface, the more likely is the corresponding combination of μ_O and μ_U in the population. In this example the (hypothetical) sample means are 3.6 for μ_O and 4.1 for μ_U. So, given the data, the combination $\mu_O = 3.6$ and $\mu_U = 4.1$ is the most plausible, or the most likely combination of values for the population means. As can be seen in Figure 2, the likelihood function achieves its maximum for this combination. Other combinations of means are less likely, for example for the combination $\mu_O = 1.5$ and $\mu_U = 2.1$ the likelihood function is much lower.

Marginal Likelihood

The third component is the marginal likelihood (M, in Table 2), which is a measure for the degree of support for each hypothesis provided by the data. The marginal likelihood is approximately equal to the average height of the likelihood function within the admissible parameter space.

Figure 1 presents the admissible parameter space for each hypothesis. Figure 2 displays the likelihood of the data. Both pieces of information are combined in Figure 3. The likelihood function in Figure 2 is presented as a contour-plot in Figure 3. The circles are iso-density contours of the likelihood function. The maximum likelihood is located in the centre of the smallest ellipse. Remember that when moving away from this centre, the likelihood of the combination of population means of μ_O and μ_U becomes smaller.

Because the admissible parameter space for H_A is equal to the complete parameter space, the marginal likelihood of H_A can be computed as the average height of the likelihood in the complete parameter space, see Table 2. This value is only meaningful in comparison to the marginal likelihood values of the other hypotheses under investigation. For H_B the average height of the likelihood function is computed on the diagonal in Figure 3. For H_C, the average height of the likelihood function is computed in the lower triangle in Figure 3. The results are presented in the first column in Table 2. As can be seen in this table, H_C has the highest value, followed by H_A and H_B, respectively.

When taking a closer look at Figure 3, we can inspect two things: model fit and model size. Consider H_C and note that the size admissible parameter space is in between hypothesis H_A and H_C. Also note that the maximum of the likelihood function in located within the admissible parameter space of H_C and H_A, but not in H_B. Moreover, the likelihood function in the upper triangle of the total parameter space, which is incorporated in H_A but not in H_C, is low. Consequently, the average height of the likelihood of H_C is larger than the average height of H_A. Hence, the marginal likelihood of H_C is larger than the marginal likelihood of H_A. For H_B the maximum of the likelihood is not within the admissible parameter space H_B. For this reason, the marginal likelihoods of H_A and H_C are larger than the marginal likelihood of H_B. Hence, the marginal likelihood rewards a hypothesis with the correct (in)equalities in the informative hypothesis, and therefore, it combines model fit and model size of a hypothesis.

Software

Software is available for: analysis of (co)variance models (Klugkist, Laudy, & Hoijtink, 2005; Kuiper, & Hoijtink, 2009), latent class analyzes (Laudy, Boom, & Hoijtink, 2005; Hoijtink, 1998, 2001), order restricted contingency tables (Laudy & Hoijtink, 2007), and multivariate linear models (Mulder, Hoijtink, & Klugkist, 2009). For more information see the edited book by Hoijtink, and colleagues (2008) or *www.fss.uu.nl/ms/informativehypothesis*.

The Results of Bayesian Model Selection

In the previous section we computed the marginal likelihood values, which can be used to compare a set of hypotheses. In this section, we show how the outcomes of the marginal likelihood can be used to calculate relative amount of support for a certain hypothesis compared to the other hypotheses. This can be done using Bayes Factors (BFs).

Bayes Factors

Bayes Factors are defined as the ratio of two marginal likelihoods (Ms). The outcome represents the amount of evidence in favour of one hypothesis compared to another hypothesis. The BF for H_C compared to H_A can be obtained from the marginal likelihoods of both hypotheses:

$$BF_{CA} = \frac{M_C}{M_A} = \frac{5.71e^{-67}}{2.83e^{-67}} \approx 2$$

This Bayes factor, BF_{CA}, implies that after observing the data, H_C receives two times more support from the data than H_A. For BF_{CB} the result implies that H_C receives

$$BF_{BA} = \frac{M_B}{M_A} = \frac{5.71e^{-67}}{1.81e^{-68}} \approx 31$$

as much support from the data than H_B.

Interpretation of the Results

When evaluating a set of hypotheses using Bayes factors, we advice the following two-step procedure.

Step 1

In the first step, the BF of each informative hypothesis is calculated against the unconstrained hypothesis. If the BF of a certain informative hypothesis versus the unconstrained hypothesis is larger than 1, it can be concluded that there is support from the data in favour of the informative hypothesis. If the BF of a certain informative hypothesis versus the unconstrained hypothesis is smaller than 1, it can be concluded that there no support from data for the informative hypothesis. The reason for calculating Bayes factors, is to inspect the overall model fit of the hypotheses under investigation. The informative hypotheses can be divided in a set of “supported” hypotheses and a set of “unsupported” hypotheses. If all informative hypotheses are considered "unsupported", the unconstrained hypothesis receives most evidence, and new informative hypothesis specification is called for. Unless a researcher is interested in which hypothesis out of a set of "unsupported" hypotheses is least unsupported.

In our simple example, H_A is the unconstrained hypothesis. The Bayes factor, BF_{CA}, for H_C compared to H_A is 2 (see Table 2), indicating that H_C receives support from the data. The BF_{BA} for H_B compared to H_A is .06 implying that H_B receives less support from the data than the unconstrained hypothesis H_A. Hence, the informative hypothesis H_B can be considered as "unsupported" while the informative hypothesis H_C can be considered as "supported".

Step 2

In the second step, we compare all the informative hypotheses under investigation with each other. From these results, it can be concluded whether the data (strongly) supports a single informative hypothesis or several informative hypotheses. For our simple example, the two informative hypotheses are H_B and H_C. These hypotheses are now directly compared with each other by calculating the Bayes factor BF_{CB}. The methodology allows for doing so, if we use the BFs against the unconstrained hypothesis, in our example, BF_{CA} and BF_{BA}. The BF_{CB} can now be calculated by

$$BF_{CB} = \frac{BF_{CA}}{BF_{BA}} = \frac{2}{.06} \approx 31.$$

This result suggests strong evidence in favour of hypothesis H_C in comparison to hypothesis H_B. So, if we were to choose between the informative hypotheses under investigation, hypothesis H_C receives most support from the data. From this analysis it can be concluded that over-controlled adolescents score lower on externalizing problem behavior than under-controlled adolescents.

Example Reconsidered

In this section we show how the informative hypotheses of the example of Van Aken and Dubas (2004) can be evaluated by first examining the descriptive statistics and applying classical MANOVA, and second, by applying BMS. Table 3 presents the observed means for all groups. We follow the two-step procedure described before to interpret the results.

Step 1. The first step involves comparing all informative hypotheses H_A, H_B, and H_C to the unconstraint hypothesis, which we shall denote by H_U. The results, which are displayed in the second column of Table 4 (BF^*), show that all informative hypotheses have a BF larger than 1 versus H_U. For example, the BF between H_A and H_U is 30.28, indicating that H_A receives 30.28 times more support than H_U. From these BFs, it can be concluded that each of the hypotheses H_A, H_B, and H_C receives support from the data.

Step 2. The second step involves comparing informative hypotheses with each other using BFs. We first want to compare H_A with H_B to decide whether additionally to the constraints of H_A, resilient adolescents function best in all psychosocial domains. The BF of H_B against H_A is given by $BF_{BA} = BF_{BU}/BF_{AU} = 64.20/30.28 = 2.12$. This result implies that the support for H_B is about 2 times as strong in comparison to H_A. From this analysis it can be concluded that additional to the constraints of H_A, there is also evidence that resilient adolescents score lower on externalizing behavior than over-controlled adolescents and resilient adolescents score lower on internalizing behavior than under-controlled adolescents as was assumed by expectation H_B.

Secondly, we are interested whether the additional constraints of H_C in (3) in comparison to the constraints of H_B are supported by the data. Consequently, the additional constraints of H_C are combined with the constraints of H_B, leading to the informative hypothesis H_C, i.e.,

$$H_C: \begin{cases} (\mu_{RHE} = \mu_{RLE}) < (\mu_{OHE} < \mu_{OLE}) < (\mu_{UHE} < \mu_{ULE}) \\ (\mu_{RHI} = \mu_{RLI}) < (\mu_{UHI}, < \mu_{ULI}) < (\mu_{OHI} < \mu_{OLI}) \\ (\mu_{RHS} = \mu_{RLS}) < (\mu_{OHS} < \mu_{OLS}) \text{ , } (\mu_{UHS} < \mu_{ULS}). \end{cases} \tag{4}$$

For this reason, we calculated the BFs of H_C versus H_A and H_B (see the fourth column in Table 4). The BFs show that there is much support in favour of H_C against H_A and H_B. For example, the BF for H_C against H_B is 21.79, stating that there is approximately 21 times as much support for H_C compared to H_B. From this analysis it can be concluded that the additional constraints of H_C are a meaningful addition to the constraints of H_B.

In conclusion, the results of BMS provide strong support for the idea that it is the combination of personality type and the quality of social relationships that puts adolescents at risk for experiencing more problem behavior.

Conclusion

In practice it is often the case that a researcher has expectations in terms of (in)equality constraints between means or regression coefficients. We refer to such expectation as *informative* hypotheses, because these include information about the ordering of the parameters. In this paper, we have shown that Bayes factors (BFs) which is a Bayesian model selection criteria, provide useful tools when determining whether such expectations are supported by the data. These selection criteria quantify the amount of support an informative hypothesis receives from the data. BFs combine the information available in the data into just one single result for each informative hypothesis taking all constraints simultaneously into account. Furthermore, we proposed an easy to use two-step procedure to interpret the outcomes of these model selection criteria. In the end, the results of BMS provide a direct answer to the research question at hand.

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Table 1.

Groups of Adolescents Based on Personality Type, Problem Behavior and Support.

		Problem behavior		
		Externalizing	Internalizing	Social
Resilient	High family support	μ_{RHE}	μ_{RHI}	μ_{RHS}
	Low family support	μ_{RLE}	μ_{RLI}	μ_{RLS}
Over	High family support	μ_{OHE}	μ_{OHI}	μ_{OHS}
	Low family support	μ_{OLE}	μ_{OLI}	μ_{OLS}
Under	High family support	μ_{UHE}	μ_{UHI}	μ_{UHS}
	Low family support	μ_{ULE}	μ_{ULI}	μ_{ULS}

Table 2.

Results of BMS for the Simple Example.

Expectation	M	BF
H_A	2.83 e^{-67}	1
H_B	1.81 e^{-68}	0.06
H_C	5.71 e^{-67}	1.99

note M = Marginal Likelihood; BF = Bayes Factor.

Table 3.

Means for the Example of Van Aken and Dubas (2004).

		Problem behavior		
		Externalizing	Internalizing	Social
Resilient	High family support (*n* = 135)	1.50	1.88	1.69
	Low family support (*n* = 70)	1.64	1.94	1.80
Over	High family support (*n* = 76)	1.43	2.05	1.77
	Low family support (*n* = 81)	1.58	2.18	1.94
Under	High family support (*n* = 70)	1.52	2.04	1.81
	Low family support (*n* = 131)	1.68	2.13	1.95

Table 4.

Results of BMS for the Example of Van Aken and Dubas (2004).

Expectation	BF*	BF**	BF***
H_A	30.28	1	46.20
H_B	64.20	2.12	21.79
H_C	1399.00	-	1

* BF compared to the unconstrained hypothesis H_U

** BF between H_A and H_B

*** BF of an informative hypothesis versus hypothesis H_C

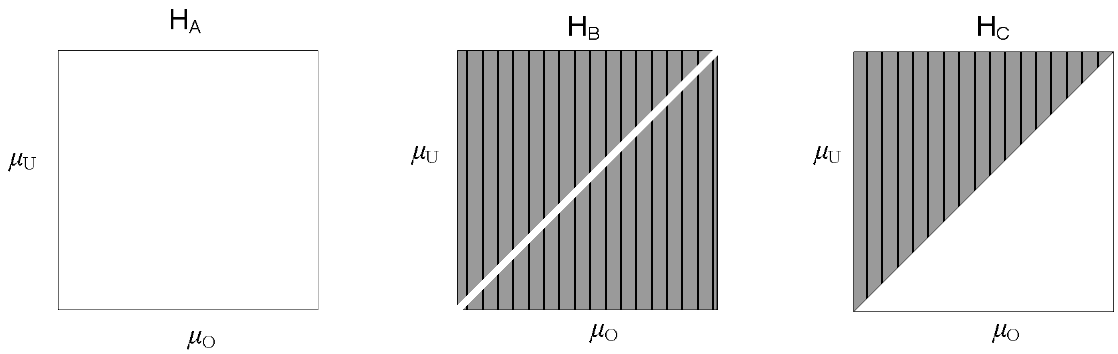

Figure 1. Admissible parameter space

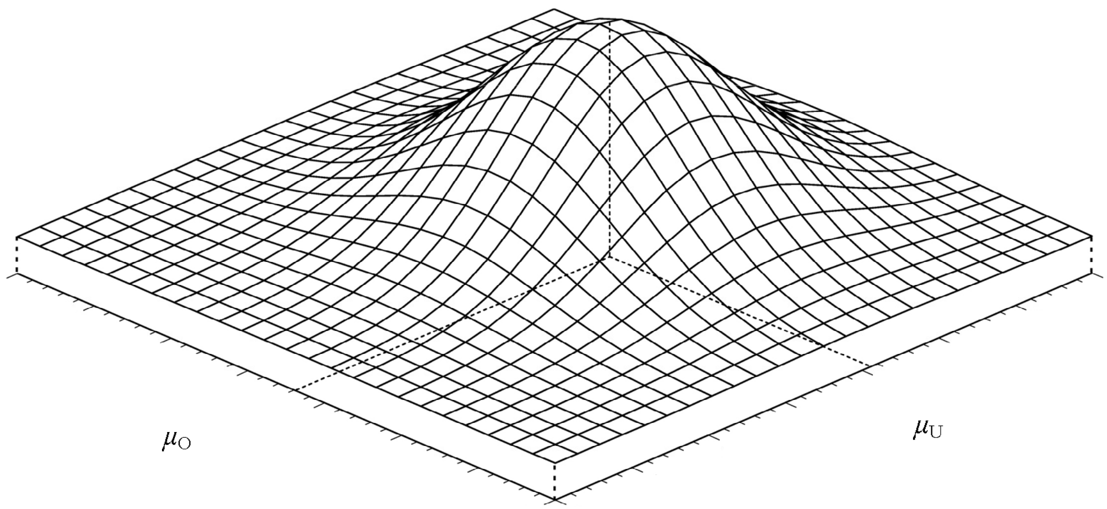

Figure 2. Likelihood of the data

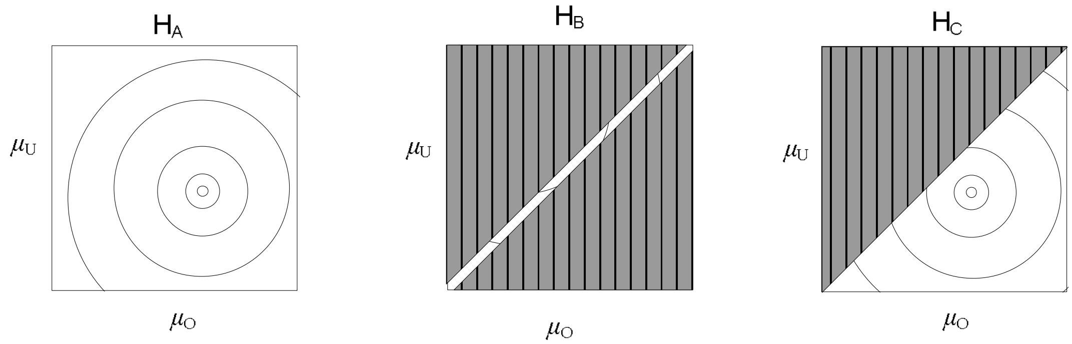

Figure 3. Likelihood of the data within the admissible parameter space