Causality Study Fortnight
Kent 2008

Intervention, underdetermination, and theory generation

⋆

Jan-Willem Romeijn
University of Groningen

Jon Williamson
University of Kent

Contents

1 Overview 3

2 Underdetermination in statistics 4

3 Factor analysis 6

4 Using intervention data 9

5 Empirical or theoretical? 13

6 Practical implications 14

1 Overview

In our paper we show that. . .

- an interesting form of underdetermination is widespread in statistical modelling within social science, specifically in exploratory factor analysis.
- a solution to this underdetermination is provided by changing the range of empirical data, specifically by employing intervention data.
- in other cases in which the intervention data do not fit, they suggest an extension of the statistical model, and thereby a change to the underlying theory.

The upshot is that aspects of scientific method that are normally associated with theoretical considerations are seen to be partly driven by empirical fact.

2 Underdetermination in statistics

Say that we want to know the ratio of people in a population that suffer from a psychological disorder D.

- We observe people indexed t to be sane or not, d_t^0 or d_t^1 respectively.
- A series of $t-1$ observed individuals is denoted s_{t-1}.
- The hypotheses are that the population has a ratio θ.

Assuming a hypothesis h_θ, any randomly drawn individual has a fixed chance of having the disorder:

$$P(d_t^1 | h_\theta \cap s_{t-1}) = \theta.$$

Any series of results is consistent with any hypothesis h_θ. But in the limit, we can almost always tell the statistical hypotheses apart.

Statistical underdetermination
This is different in the following statistical study, characterised by a different set of statistical hypotheses, g_ξ:

$$P(d_t^1 | g_\xi \cap s_{t-1}) = \xi^2 \qquad \xi \in [-1, 1].$$

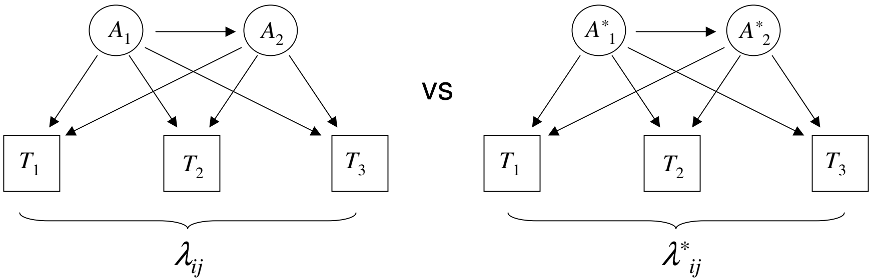
The hypotheses g_ξ in this statistical problem are essentially the same as the h_θ, only they are labeled in a funny way. For any value $\nu \in [0, 1]$ we have

$$P(d_t^1 | g_\nu \cap s_{t-1}) = P(d_t^1 | h_\nu \cap s_{t-1}) = P(d_t^1 | g_{-\nu} \cap s_{t-1})$$

Hypotheses g_ν and $g_{-\nu}$ are indistinguishable, because they both assign exactly the same probability to all the observations. In such a case, we speak of an unidentifiable model.

3 Factor analysis

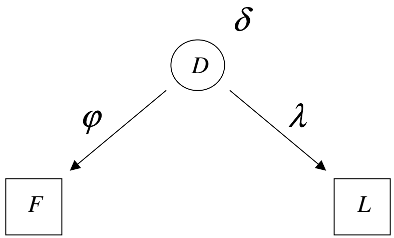
There are more realistic cases in which such unidentifiable models occur. One example is the so-called rotation problem in factor analysis, a technique that is widely used in the social sciences.



The rotation problem is that there are infinitely many ways to regress a set of correlated manifest variables on a set of latent variables.

Fear and loathing in Bayesian networks

Say that fear F and loathing L are both binary manifest variables, and consider a single latent cause, depression D. Observations are of individuals t having f^j and l^i.



Every probability assignment consistent with the graph is a hypothesis h_θ. In other words, the hypotheses are Bayesian networks on the given graph.

Underdetermination in Bayesian networks

We may count the degrees of freedom in the statistical model, after applying a fit criterion. We have a total of five parameters in the statistical model:

$$P(d_t^1 | s_{t-1} \cap h_\theta) = \delta \,,$$
$$P(f_t^1 | d_t^k \cap s_{t-1} \cap h_\theta) = \phi_k \,,$$
$$P(l_t^1 | d_t^k \cap s_{t-1} \cap h_\theta) = \lambda_k \,.$$

But we have only three observed relative frequencies, namely for the observations

$$f_t^j \wedge l_t^i \quad \text{with } i, j \in \{0, 1\},$$

to fill in these values. An infinity of statistical hypotheses therefore fit the data perfectly.

4 Using intervention data

The basic idea of using interventions for the purpose of solving underdetermination is that...

- an intervention changes the distribution over the latent variable of the subjects. In the example, the probability for depression δ is altered.

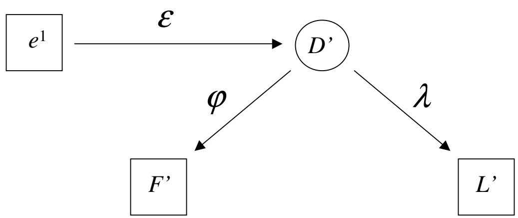
- it does not change the probabilistic relations between the latent and the manifest variables. In this case, we keep the probability of fear and loathing conditional on depression, λ_k and ϕ_k, fixed.

- after the intervention, we obtain a new estimation problem for the parameters in the statistical model.

The key point is that to accommodate the intervention data, we have a smaller space of parameters available.

Drugs to the rescue

Say that we intervene on the depression by administering a drug E to the subjects. We model this by an additional node, setting the probability for depression to a new but unknown value ϵ.



Note that in order to frame the intervention, we assume that the latent variable model is correct and that we intervene only on the depression node.

Underdetermination resolved

Because we observe the fear and loathing of the individuals after the intervention, we have three additional observed frequencies.

$$f_t'^i \wedge l_t'^j \quad \text{with } i, j \in \{0, 1\},$$

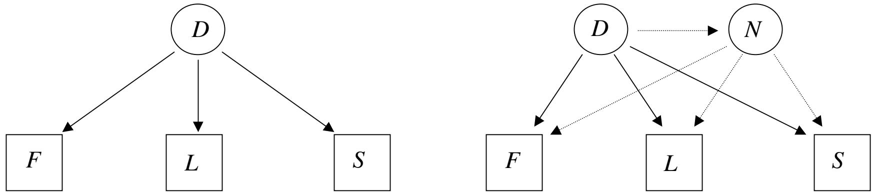
So we have six relative frequencies in total. But we have only one additional parameter in our problem, namely the probability for depression after taking the drug,

$$P(d_t^1 | e_t^1 \cap s_{t-1} \cap h'_{theta}) = \epsilon,$$

bringing the total number of parameters in the hypothesis h'_θ on six as well. Consequently, there is only a single solution of the parameters that fit the data perfectly!

Drugs as a guidance to theory change

It may also happen that the fit with the collected observational and interventional data is poor, according to some model selection or fit criterion.



In that case we can add latent variables to the model, enlarging the statistical model. The intervention data suggest specific ways of doing this. They guide the conceptual change.

5 Empirical or theoretical?

Thus empirical considerations replace the theoretical considerations that are normally used to resolve underdetermination and drive theory change.

- The rotation problem is typically resolved by an assumed independence structure among the latent variables, or the assumption of maximal variation among the regression coefficients.

- Changes to the statistical model or patterns are usually left to the creative imagination of the researcher.

But the interventions are not entirely empirical: the way in which they impact on the underdetermined or overdetermined latent structures depends on a causal picture that is assumed beforehand.

6 Practical implications

We think that the use of interventions will be of practical use in the sciences.

- The problem of rotation is not merely academic. It affects theories and tests in economics and psychology. Interventions may help to resolve real underdetermination in these sciences.

- Model selection is a live issue in the application of statistics in the sciences. The generation of extensions of the model by intervention data can be used to improve selection procedures.

The ideas may also be used in the philosophy of science. Confirmation has been studied extensively with formal means. With the present study, we hope to contribute to a formal philosophy of experiment.

Thank you

The slides for this talk will be available at http://www.philos.rug.nl/ romeyn and the full paper is also posted there. For comments and questions, email j.w.romeijn@rug.nl.