Reasoning with probabilities

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Reasoning with probabilities

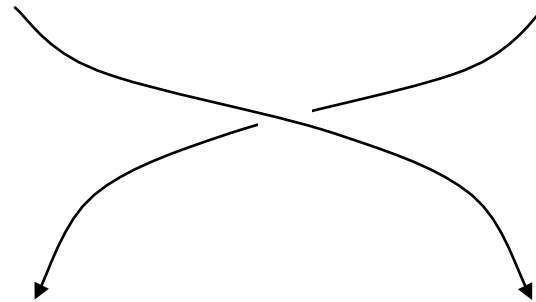Sound statistical reasoning is important for everyone.

Lucia de B.

The initial argument in court ran something like this.

The chance of the incidents assuming she is innocent is low

Therefore,

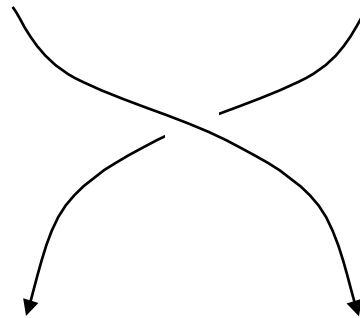*The chance that she is innocent given the incidents is low*

Lucia de B.

The initial argument in court ran something like this.

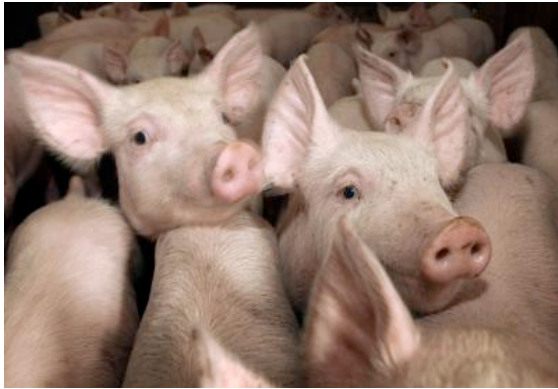$$\text{Chance (incidents | she is innocent) is low}$$

Therefore,

$$\text{Chance (she is innocent | incidents) is low}$$

Is this reasoning sound?

Swine fever

We encounter the same reasoning in science.

Chance(test positive|not ill)
is very low

Chance(not ill|test positive)
is also very low

Intelligent design

And also in the debate over evolution and ID.

Chance(ordered nature|no ID)
is very low

Chance(no ID|ordered nature)
is also very low?

Correct reasoning

What to think of this inference?

$Chance(win|no\ fraud)$
is very low

$Chance(no\ fraud|win)$
is not low at all!

Bayes' rule

Thomas Bayes proved in 1763 that reasoning in this way requires more input.

$$Chance(hypothesis|data)$$
$$=$$
$$\frac{Chance(data|hypothesis)}{Chance(data)}$$
$$\times$$
$$Chance(hypothesis)$$

Bayes' rule

Thomas Bayes proved in 1763 that reasoning in this way requires more input.

$$Chance(\textcolor{blue}{hypothesis}|\textcolor{red}{data})$$

$$=$$

$$\frac{Chance(\textcolor{red}{data}|\textcolor{blue}{hypothesis})}{Chance(data)}$$

$$\times$$
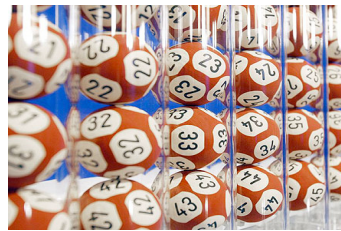
$$Chance(hypothesis)$$

Prior probability

In our examples *Chance*(*hypothesis*) is crucial.

Chance(*not ill*) ?

Chance(*no ID*) ?

Chance(*no fraud*) ?

Chance(*innocent*) ?

Conclusion

We cannot invert the probability without further assumptions.

Always keep Thomas Bayes in mind!