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Frequencies, Chances and Undefinable Sets

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What this talk is about

Chances play an important role in science, especially in the behavioral and social sciences.

Psychology: Latent psychological attributes are tied to psychological test by chancy relations.

Sociology: Variation due to individual behaviour is captured by describing groups in stochastic terms.

Economics: Trends are described by time-series that include a systematic component and chances.

What are chances?

Chances are described by probability distributions. But how can we interpret and use these distributions?

- What is the role of these distributions in inference?
- How do probability distributions relate to the world out there?

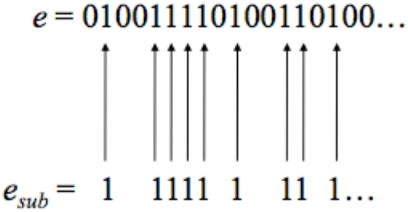
In response to this, I argue that

- hypotheses on chance have a definite function in the formal semantics of statistical inference, and that
- they are objectively true or false because chances can be interpreted as objectively determined properties.

Contents

1 Von Mises’ frequentism

An important starting point in the discussion on chance is the frequentist definition of probability by von Mises.



It is based on the empirical notion of a collective, defined by a limiting relative frequency and the exclusion of a so-called gambling system.

Problems with frequentism

Frequentism has been criticised heavily.

- ↯ Infinite sequences: in the long run, we are all dead (Keynes).
- ↯ Random sequences: seemingly intentional aspects to place selections (Ville, but see van Lambalgen).
- ↯ Finite frequentism: a host of problems (Hajek).

I do not want to claim that von Mises cannot defend his theory against all these criticisms. Still...

Alternative frequentisms

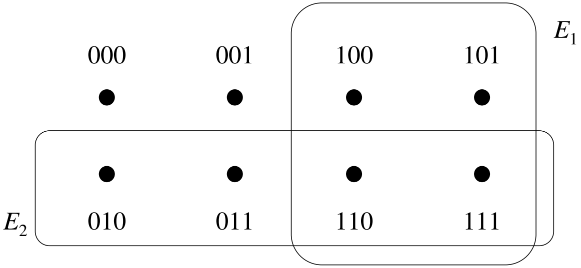
In this talk I employ the frequentist ideas for other aims than for an empiricist theory of chance.

Semantic frequentism provides a formal semantics for statistical inference by specifying the nature of statistical hypotheses.

Metaphysical frequentism fosters an interpretation of single-case chances that escapes the reference class problem.

2 Frequentist statistical hypotheses

A statistical analysis is always based on a set of possible observations, a sample space. For tossing a coin N times, the sample space is $\{0,1\}^N$.



Samples e_t can be represented as sets E_t in this space.

Hypotheses as distributions

We may also construct an idealised sample space consisting of infinitely long samples: $\{0,1\}^{\Omega}$.

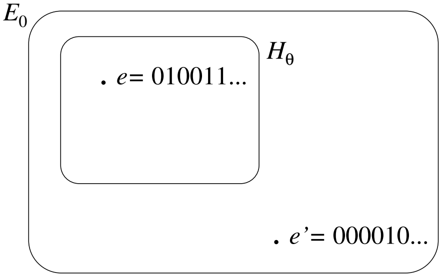
$$e = 010011011001010\dots$$

$$P_{\theta}(E_t|E_{t'}) = \theta$$

We can define the statistical hypothesis h_{θ} as a distribution over this infinite sample space.

Hypotheses as tail events in the algebra

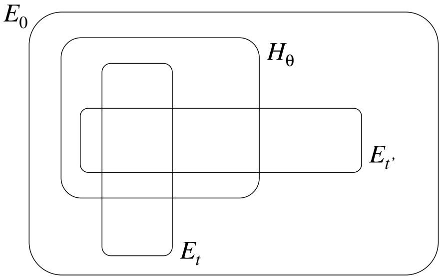
Some elements $e = 00110111...$ of this sample space are collectives in the sense of von Mises.



We can identify statistical hypotheses h_θ with the set of all collectives H_θ that, according to frequentism, instantiate the probability distribution P_θ.

Tail events as distributions

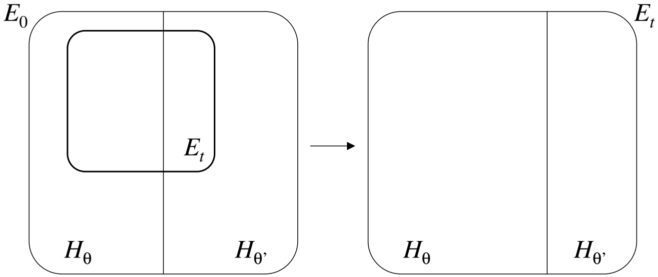
Each set H_θ intersects with all sets E_t that are assigned nonzero probability by P_θ.



The hypothesis h_θ, conceived as a probability distribution, is thus connected to a quasi-observational set H_θ.

3 Frequentism in Bayesian inference

The principle of Bayesian inference is conditionalisation. Data e_t are reflected in the probability assignment by zooming in on the probability assignment within e_t: $P(E_{t'}) \rightarrow P(E_{t'}|E_t)$.



We can apply this idea to hypotheses as sets in sample space.

Hypotheses fix inductive dependence

In Carnapian inductive logic, the inductive influence of observations e_t on expectations over other observations $e_{t'}$ is fixed directly, by means of inductive rules.

Sets of statistical hypotheses, or models for short, provide a convenient way of fixing this inductive dependency. Tail event hypotheses extend the language of inductive logic.

4 Frequentism as formal semantics

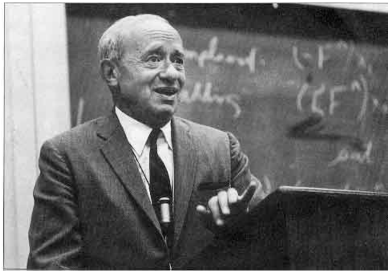
The first aim was to specify the nature of statistical hypotheses and their role in statistical inference.

- The statistical hypotheses are interpreted in the idealised setting of an infinite sample space.
- As such, the hypotheses perform a definite function in a logic of statistical inference.

We briefly comment on this intermediate conclusion.

Formal semantics for statistics

The success of deductive logic is partly based on a clear separation of syntax and semantics, which led to notions such as validity, truth in the model, etc.



The above formal semantics are a step towards putting inductive logic on the same footing as deductive logic.

Formal semantics for statistics (continued)

We have sketched a formal semantics for statistical inference. We now start on some model theory and logic.

Soundness: if the inference machinery proves a proposition, it is guaranteed to be true in the model.

Completeness: if a proposition is true in the model, the inference machinery is guaranteed to prove it.

For Bayesian statistical inference we can reformulate completeness as convergence.

Formal sematics for statistics (continued)

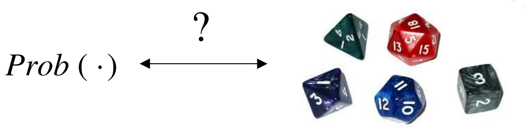
The convergence theorem by Gaifman and Snir may be used as completeness theorem.

> **Completeness:** If a hypothesis is true, if we do not assign zero probability to it, and if the data separate the hypotheses, then in the long run Bayesian statistical inference will give it probability 1.

Soundness fails because the initial choice of hypotheses may lead us astray.

Reversed frequentism

Von Mises presented frequentism as a theory on what probability is, empirically grounding it in mass phenomena.



Instead, a notion of probability is here presupposed, and frequentism is used to relate it to idealised empirical fact.

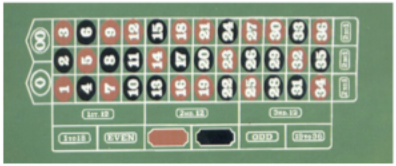
Reversed frequentism (continued)

The problematic notion of collective is taken as part of a formal semantics, and thus of model theory. Most of the criticisms against von Mises do not apply.

We find a semantic role for frequentism within an epistemic, either subjective or objective, interpretation of probability.

5 Reference classes and reductionism

We have located chances firmly within the epistemic domain, as components of a formal semantics for statistics.



What about chances as pertaining to physical systems, that is, as elements of reality?

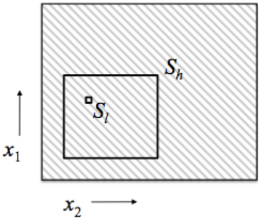
Reference class problem

The central problem for any notion of objective probability is the reference class problem: different descriptions of the same individual event lead to different chances.

Chances thereby become description-relative, and thus subjective rather than objective.

Reductionism and deterministic systems

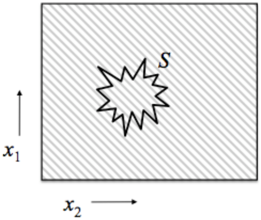
A special case of the reference class problem presents itself for deterministic systems: a complete description of such systems trivializes their chances.



Chances thus become essentially epistemic: their value is given by our lack of information on the deterministically determined states.

6 Irreducible chances and frequentism

We can use the idea of multiple realizability to define a concept of chance that escapes the problem of the reference class, and hence of reductionism.



It may so happen that the macro-level description of a physical state cannot be reformulated in terms of sets of micro-level states.

Undefinable sets of states

The key to this impossibility of reformulation is in the definabilty of the macro-state in terms of micro-states.

$$S = \{x : f_S(x) = 1\}$$

If the characteristic function $f_S(x)$ is not recursively definable, then we cannot conceive of the macro-state as a coarse-grained version of the micro-state.

Classic comeback: fundamentality

It may be argued that the macro-level desciption is nevertheless derived from the micro-level. This amounts to the metaphysical position that composites are ontologically prior.

But micro- and macro-states are mutally undefinable: macro-states may just as well be considered fundamental.

Frequentist underpinnings

There is a formal connection between frequentism and this anti-reductionist conception of chance. For any recursively definable set of micro-states C_i,

$$P(E|S \cap C_i) = P(E|S).$$

This is formally similar to the absence of a recursively definable subsequence within a collective for which the limiting relative frequency deviates from the original collective.

7 Conclusions

I have argued that the frequentist theory of chance can be used to our advantage in two separate philosophical projects.

- It provides a formal semantics for statistical inference by specifying the nature of statistical hypotheses.
- It fosters an interpretation of single-case chance that can escape the reference class problem and reductionism.

Ironically, the theory was motivated by strict empiricism but seems to find promising applications in clarifying metaphysical and theoretical notions.

Thanks!

This talk will be available at `http://www.philos.rug.nl/~romeyn`.
For comments and questions, email j.w.romeijn@rug.nl.