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What is a counterexample?

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Formal philosophy

There is a long tradition of formal analysis in philosophy, certainly insofar as it is concerned with matters epistemic.

Logic provides a formal account correct inference.

Confirmation theory formalizes the support relation between theory and evidence.

Belief revision theory presents a formal structure for belief maintenance.

Of course the list goes on. . .

The informal world

The informal world of the epistemic is best described by models from the social sciences. By contrast, philosophical models. . .

- take a normative stance and hence motivate action,
- concern ideal agents rather than real people, and
- are often tested against intuitions rather than empirical facts.

Despite the differences, there is a trend to bring social science and formal philosophy together.

Model vs world

What to conclude if formal models do not square with our intuitions or with empirical fact? Are the models wrong, or applied incorrectly?

This choice has implications for the role and status of formal philosophical models in the social sciences.

Contents

1 Normative and descriptive models

There are many interactions of formal models in philosophy and social science:

- Normative formal models are there for real people to aspire to. Social science can inform philosophers on how to tailor and communicate their models.
- Social science theories need conceptual tools and foundations. Philosophy can provide formal structures for unifying and interpreting empirical findings.

Experimental philosophy

Empirical facts about people's reasoning and ethical considerations routinely informs modelling in philosophy. Examples:

- The psychology of reasoning has stimulated philosophical research into alternatives to the deductive consequence relation.
- The linguistics of conditional sentences impacts on the philosophical debate over the semantics of conditionals.
- The economics and psychology of decision making has put pressure on philosophical models of practical rationality.

Foundational input

Conversely, normative philosophical models have often been used to perform a descriptive function in the sciences:

- Probabilistic degrees of belief have been employed descriptively in a wide range of disciplines.
- Epistemic logics have been used to describe strategic economic behavior.
- Formal philosophical ideas on causality have made their way into cognitive psychology of causal inference.

Model failure or misapplication

For meaningful interactions we need to know in what sense the normative models can be wrong.

If every failure can be put down to misapplication, the model cannot be falsified.

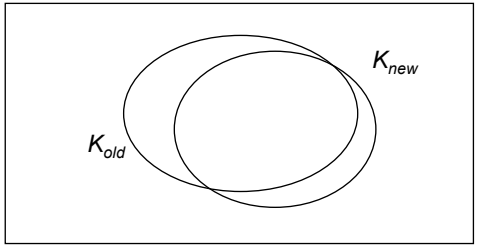
The role of the model

The model is in that case not a substantive theory on norms or facts, but perhaps a good framework for theorizing.

The model might serve as a universal tool to identify the presuppositions of real, or ideal, reasoning.

2 Iterated belief revision

AGM belief revision is a theory on revising beliefs in the light of evidence: new evidence can be accommodated by expanding, contracting, or shifting the set of sentences currently entertained as true.



The theory is diachronic and qualitative and thus fits well with the coarse-grained and dynamic nature of real reasoning.

Iteration

The theory runs into unintuitive consequences when the rules for revision are applied iteratively. One source of problems is *meta-information*:

> . . . information about how I learn some of the things I learn, about the sources of my information, or about what I believe about what I believe and don't believe. If the story we tell in an example makes certain information about any of these things relevant, then it needs to be included in a proper model of the story, if it is to play the right role in the evaluation of the abstract principles of the model.
>
> Robert Stalnaker (2009), Iterated Belief Revision, *Erkenntnis* 70, pp. 189–209.

Problematic postulates

The following postulates are often added to the basic AGM postulates that determine the revisions:

I1 demands that if $\phi \rightarrow \psi$ is a theorem (with respect to the background theory), then first learning ψ followed by the more specific information ϕ is equivalent to directly learning the more specific information ϕ.

I2 demands that first learning ϕ followed by learning a piece of information ψ incompatible with ϕ is the same as simply learning ψ outright, e.g., first learning ϕ and then $\neg\phi$ should result in the same belief state as directly learning $\neg\phi$.

Counterexample to I1

There are three switches, xyz. The light U is on if $x = y$, and W is on if $x = y = z$.

Alice, Bob, and Carla report respectively that switch $x = 1$, $y = 0$, and $z = 1$.

Light U is observed to be on.

Light W is observed to be on.

After this series of reports, belief revision prescribes that we believe that xyz are 111. But if we directly learn that light W is on, we may also hold 000 possible.

Counterexample to I2

Two fair coins are flipped and placed in two opaque boxes, 1 and 2. Independent and reliable observers deliver reports about their status, heads or tails up, H_i or T_i.

Alice and Bob report that the coin in respectively box 1 and 2 are heads up, H_1 and H_2.

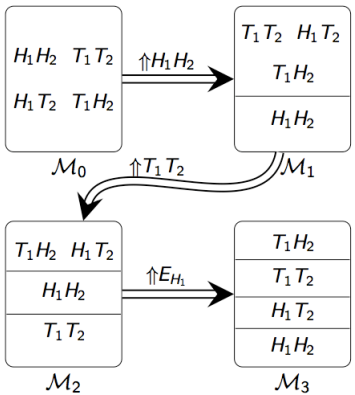
Carla and Dora report that the coin in respectively box 1 and 2 are tails up, T_1 and T_2.

Elmer reports that the coin in box 1 is heads up, H_1.

After this series of reports, belief revision prescribes that we also believe that H_2.

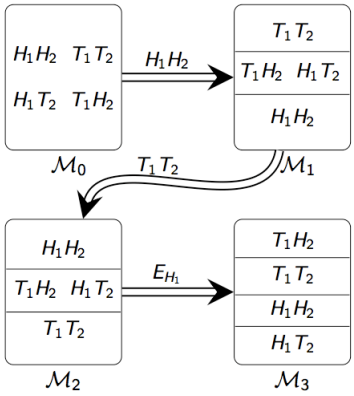
Diagnosis of the latter

The ideal agent comes to believe that the coins are correlated, but this meta-information is not explicit in the story. It is a side effect of learning first that both coins are heads and then that they are both tails.



Repairing belief revision

After every learning experience we can retain the initial belief that the position of the coins are independent. What is the belief revision *policy* generating this sequence?



3 Lexicographic Bayesian models

We accommodate the counterexamples in two steps:

- We provide a Bayesian model in which presuppositions on order and dependence can be made explicit.

This model already satisfies the basic postulates of belief revision.

- The qualitative and diachronic character of belief revision can be replicated by an extension to lexicographic probability assignments.

Apart from this we refined the event structure of reports and states.

Highly flexible models

The Bayesian model enriches the range of doxastic attitudes that the ideal agent can have.

In this way we can model all manner of meta-information. AGM belief revision can be made to fit counterexamples to both I1 and I2.

4 Discussion on counterexamples

We can draw some general lessons about the balance between counterexamples and misapplications of formal models of belief dynamics, concerning. . .

- the generality of belief revision theory,
- its criteria for applicability,
- the use of including meta-information in the model, and
- the existence of genuine counterexamples.

Criteria for application

Notice that AGM belief revision does not come equipped with its own user manual. The criteria for application are extra-theoretical.

It is therefore hard to say how far the theory might stretch in general.

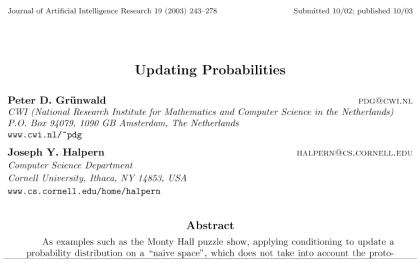
Don’t be lazy

The Bayesian resolutions illustrate that belief revision can accommodate particular kinds of meta-information, pertaining to the conceptual, causal, and epistemic relations among information items.

Notably, the fact that we do not find a model, does not entail that there are none.

Coarse-grained modelling

In some examples we can ignore the meta-information, which is often not specified in the description of an example.



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Updating Probabilities

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Abstract

As examples such as the Monty Hall puzzle show, applying conditioning to update a probability distribution on a "naive space", which does not take into account the proto-

For Bayesian models, Halpern and Grünwald define the condition of *coarsening at random*, which holds when a more coarse space of reports yields the same belief dynamics.

Genuine counterexamples

Conversely, genuine counterexamples to belief revision are cases in which no amount of refining events, reports, and doxastic attitudes will help.

We think that Grünwald and Halpern lead the way to a systematic characterization of counterexamples.

5 Back to the informal world

It is an open question whether current formal philosophical models have any genuine counterexamples in the above sense.

But if they do not, the models may still offer great value to substantive social science.

Coordinate principles

Geometry cannot by itself be falsified by measurements of physical space. To apply it to space, we need coordinative principles.

The normative formal models may well have exactly that status in social science: they establish structure but we cannot prove them wrong.

Thank you

The slides for this talk will be available at http://www.philos.rug.nl/~romeyn.
For comments and questions, email j.w.romeijn@rug.nl.