IPSP 2014 at LMU

Statistics and Full Belief

Or: How to Judge if You Must

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Ring-ring

You're called up by a decision maker who says: what statistical facts can I rely on?

What do you report? Can you commit to your favourite statistical hypothesis?

This talk

We argue the following.

- The problem of how statistics relates to full belief is in need of a principled solution.
- One attractive solution employs a notion of belief based on stability, as developed by Leitgeb.
- This solution illuminates how full belief is sensitive to the context of the decision maker.

Contents

1 Beliefs in statistics

In the foundational debates, classical and Bayesian statistics offer only limited room for a notion of belief.




Both sides work from empiricist and behaviorist presuppositions. Central notions are decision and action.

Belief vs decision

There is a pragmatic reason for accommodating belief in a statistical methodology.


(Concept)

ZonMw

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Begrippenkader

'Gepaste zorg en praktijkvariatie'

Informatie ten behoeve van de **tweede Invitational Conference**

op **18 juni 2014 (13:00–17:30) in de Domus Medica** te Utrecht

Often a decision is political or ethical, rendering a decision-theoretic treatment infeasible. A statistical procedure should then be able to offer epistemic guidance.

Radical probabilism?

Belief may sensibly be conceived and reported purely probabilistically. However. . .

- Scientists encounter practical problems in communicating finnicky probability assignments.
- People may not want to commit to probability assignments as an expression of their beliefs.

So there are reasons for accommodating full belief as well.

2 Stable belief

How to introduce full belief states into statistics? Some earlier attempts:

- Kyburg's logic of defeasible commitments to statistical hypotheses.
- Mayo's attempts to epistemicize classical statistics, primarily Neyman-Pearson testing.
- Levi's partition-sensitive acceptance rules as add-on to Bayesian analyses.

We are close to Levi's approach: full beliefs will appear to be context-sensitive.

Relations to statistics

Classical and Bayesian statistics relate to categorical belief in complementary ways.

- Classical statistics relates naturally to categorical judgments but not in the format of belief.
- Bayesian statistics relates naturally to belief but not in the categorical format.

We aim for a middle ground: categorical beliefs have a systematic relation to interval-valued probability.

Stability as a guideline to acceptance

Leitgeb developed a systematic non-reductive relation between categorical and probabilistic beliefs, based on a notion of stability.

> *Stability-based full belief*
> A probability assignment warrants commitment to a categorical belief in a proposition X iff for any proposition $A \in \mathcal{A}$ we have $P(X|A) > r$, where $\mathcal{A}$ is the set of propositions whose negation is not included in the full belief state.

The idea of stability-based belief is reminiscent of Skyrms's theory of chance. But there are other ways of arriving at the same belief concept.

Properties of stability-based beliefs

We can reverse the definitional order: a full belief state generates, or can be represented by, a credal set.

- The full belief state respects Hintikka's doxastic logic and Lehmann and Magidor's belief revision theory.
- It has an appealing semantics of Grove spheres, so that the corresponding constraints on probability assignments are easily computed.
- It does not take either mode of expressing beliefs, categorical or probabilistic, to be more fundamental.

3 Application to statistics

Our proposal is to apply this belief conception to statistics, to offer advice to our decision maker.

Consider a surgeon who asks our opinion on a null hypothesis and who will incorporate one additional factor in her decision.

A simple example

We may re-represent Neyman-Pearson testing as an inference by adopting a coarse-graining of the sample space, R^1 vs R^0 and a threshold acceptance rule $P(H_0)|R^1) < t$, so that we can derive

$$P(R^1|H_0) = \alpha,\ P(R^1|H_1) = 1 - \beta \quad \Rightarrow \quad P(H_0) \in [l, u].$$

We can thus reconstruct an interval-valued prior from the required categorical outcomes of the statistical inference.

From beliefs to statistics and back

The stability-based notion of belief offers a systematic way of reconstructing credal sets from full belief states, and conversely.

- A classical statistician who takes H_0 as default, and defeasibly attaches full belief to R^0, can be represented with a particular credal set. This follows from natural consistency constraints on her epistemic commitments.
- A Bayesian statistician whose probability assignment falls within a particular credal set is warranted to commit to the full beliefs associated with that credal set.

Exploiting the bridge principles

We may adapt the setup of our statistical tests to the constraints that are dictated by the required full belief outcomes. The credal sets associated with these outcomes determine these constraints.

- What sample size will warrant the report of a full belief, given a decision context?
- Which tests, and hence values of α and β, are needed for reaching the requisite full belief conclusions?
- How do the full belief states depend on the threshold value r, and how do our commitments change if we raise the threshold?

The Cartesian surgeon

The set of full beliefs depends on threshold, test specifics, priors, and crucially, the set of possibilities $\mathcal{A}$.

We adapt our report to the surgeon according to what is salient to her decision situation. This might even nullify our full belief set.

4 Contextual beliefs

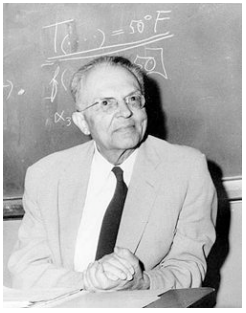
We think it fitting that the reported full beliefs depend on what propositions are deemed salient by the decision maker.

- In classical statistics, acceptance and rejection are relative to the choice of a sample space.
- In Bayesian statistics, acceptance rules are often taken to be relative to a partition of the algebra.

Full belief is often deemed contextual or even “holistic”.

In terms of “isms”

The context sensitivity of belief mirrors the dependence of decisions on context.



We shed behaviorism and strict empiricism but we retain pragmatism.

5 Summing up

We argued for the following.

- The problem of how full belief relates to statistics is in need of a principled solution.
- An attractive solution employs a notion of belief based on stability, as developed by Leitgeb.
- This solution illuminates how full belief is sensitive to the context of the decision maker.

Future work

We will employ this view on statistics and belief in an analysis of various statistical techniques.

- Derivation of context-sensitive constraints on reporting hypothesis tests.
- A similar analysis of confidence intervals using the closely related credal intervals.
- Robustness analysis of models via full beliefs based on Bayesian model selection.

Thank you

The slides for this talk will be available at http://www.philos.rug.nl/ romeyn.
For comments and questions, email j.w.romeijn@rug.nl.