Chance encounter

Schnorr Randomness and Autonomous Chance

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Jan-Willem Romeijn
University of Groningen

The reference class problem

The chances for the individual events $a(t)$ can be defined as the proportions of the event types A in a class of similar events C within a space X.

- Assuming determinism, a full specification of the event, i.e., using a singleton C, leads to trivial chances.
- By choosing to characterize the reference class C narrowly, we can tweak the chance ascriptions at will.
- Sometimes we must choose between rivaling reference classes C and C', i.e., when we do not have information concerning $C \cap C'$.

Model selection

The problem may seem far-fetched but it is actually central to the proper use of statistical methods in policy and science.

- When is statistical evidence appropriately used in the court case of a particular individual?
- How can we identify patient groups that respond differently to medical treatment programmes?
- What variables are salient when determining the risk profile of an insurance portfolio, or the tendency towards crime of disadvantaged adolescents?

This talk

I aim to argue the following.

- Some reference classes provide a level-relative yet objective basis for a chance ascription.
- These level-relative chances are autonomous, i.e., they are independent of chance ascriptions that obtain on other levels of description.
- We may then speak of objective chances, even though the events to which the chances are ascribed are fully determined.
- We have reason to believe that such autonomous chances exist.

Several others have argued for similar theses, e.g., on emergent chance. My news is mostly in the use of von Mises' frequentism.

Contents

1 Probabilistic autonomy

We can express the idea of autonomous level-relative chance by means of probabilistic independence. Chances are autonomous relative to some algebra $\mathcal{B}$ if for all refinements B_i from this algebra we have

$$P(A|C \cap B_i) = P(A|C).$$

The independence indicates that adding further information from $\mathcal{B}$ to a reference class C does not affect the chances for A. Other terms for the same idea are the robustness and resilience of chance.

Autonomy (continued)

The ultimate version of such independence has the events A distributed uniformly over the space X. Consider the σ-algebra $\mathcal{X}$ generated by all sets of the form

$$B[x, \delta] = \{x' : x' \in (x, x + \delta)\}$$

where the δ can be arbitrarily small, and stipulate that

$$P(A|B[x, \delta]) = P(A).$$

Even the most fine-grained algebra does not offer additional information on the event A.

Autonomy (continued)

In the above setup each point x is labelled with either 0 or 1, that is, there is an indicator function $f_A(x)$ that determines set membership for A:

$$f_A : x \mapsto \{0, 1\}$$

is a well-behaved function. But this function is not definable with the conceptual means offered by $\mathcal{X}$. The function f_A is infinitely intricate.

2 Von Mises' place selections

The idea of infinite intricacy can be spelled out by means of Von Mises' notion of a gambling system. A binary series A may or may not have a stable limiting relative frequency R of outcomes:

$$R(A) = \lim_{n \to \infty} \frac{\Sigma_{t=1}^{n} a(t)}{n}$$

where $a(t)$ is the digit at position t in the series.

Von Mises (continued)

We can use a second series B_i to select elements from the series A, and construct the relative frequency of a subseries:

$$R(A;B_i) = \lim_{n\to\infty} \frac{\Sigma_{t=1}^{n} a(t) b_i(t)}{\Sigma_{t=1}^{n} b_i(t)}.$$

We can repeat this place selection with a series C.

Von Mises (continued)

The series A is random relative to a set $\mathcal{B}$ of such selection rules if

$$R(A) = R(A; B_i)$$

for all B_i in it. Notice that we can view the series A and B as sets of natural numbers, and view the relative frequencies as probability assignments of sorts. We then arrive at a version of probabilistic independence.

3 Random events

We can apply the same notion of place selections in the richer context of events A, $B[x,\delta]$, and C. We employ the mathematical machinery of place selections, or statistical tests, in specifying the aforementioned infinite intricacy of the event A.

- Randomness is always relative to a class of place selections. In this case, it is relative to an algebra $\mathcal{X}$ over the space X.

- The place selection may not be based on the series itself. In our case, the event A may not be definable in the algebra $\mathcal{X}$.

- A forteriori, for any $B[x,\delta]$ we must have that $P(A|B[x,\delta]) = P(A)$.

Random events (continued)

Parallel to the binary series, we say that a chance ascription $P(A)$ is objective if the event A is random relative to $\mathcal{X}$. Some remarks:

- There are many variations on the randomness of the event A depending on the details of the algebra $\mathcal{X}$. This runs parallel to the randomness of sequences.
- A natural line is drawn by the algebra $\mathcal{X}_S$ that corresponds to "Schnorr randomness": whether or not a point x is a member of an element $X \in \mathcal{X}$ must be computable.
- This leaves room for the random event A to be the result of an effective procedure.

Random events (continued)

- We have omitted the reference class C and oriented on the entire space X in the foregoing. It can be reintroduced by noting that all actual chance processes manifest in a countable infinity of trials.
- Importantly, every point x in the space X, or in the class C, is labelled as either $f_A(x) = 0$, or $f_A(x) = 1$. We need not violate determinism.
- We nevertheless obtain autonomous chances for A, relative to a macroscopic reference class C and an algebra $\mathcal{X}$.

4 Objective single-case chance

Do we have reasons to believe that these formal constructions map onto anything that exists in our world? The question is eventually an empirical one. Some considerations:

- Events and reference classes may be generated mechanically and yet be infinitely intricate. The surprising complexity of fractals are a case in point.
- The idea that levels of description cannot be related in a neat way is well-known from the reductionism debate: states may be multiply realized in a way that is not finitely axiomatizable.

Thank you

The slides for this talk will be available at http://www.philos.rug.nl/ romeyn.

For comments and questions, email j.w.romeijn@rug.nl.